# Steering Information Cascades in a Social System by Selective Rewiring and Incentive Seeding

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***Abstract*—Information cascade characterizes the situation where rational agents are overloaded with social observations and can make decisions ignoring their own personal evidence. If the social observations are misleading, the overall system performance can be subsequently compromised. Therefore, it is of interests to design a socially or interactively networked system falling into undesirable or desirable cascades. In this paper, we study the driving factors of steering information cascades in a socially networked system by considering a sequential Bayesian social learning model with arbitrary network topology and utility function. In particular, we analyze the impacts from different random network topologies on the overall system performance such as error probability, fraction of cascades, and total utility. Furthermore, we demonstrate two systematic approaches, selective rewiring and incentive seeding, which can change the information structure observed by agents and hence steer information cascades. Our results show that when agents restructure their observation network and provide proper incentives for cascades in the correct decisions, the overall social system performance can be well-steered.**

## I. Introduction

With the proliferation of Internet social networking services in modern society, it becomes more and more important to understand how agents respond to incentives and make decisions in social networks, when engineers and social planners build large-scale social systems which manage the coordination and control of complex user behavior and dynamics. For example, a customer review system in e-commerce, where customers can browse the reviews of previous customers before making their decisions to buy products. From customer's perspective, each customer aims to determine whether it is a good decision to buy the product. Customers can rely on the information provided by the review system to learn about the product. On the other hand, customers can also proceed some research on the product by themselves, i.e. searching other websites to get more information, asking expert opinions, or judging the product by personal experience.

In this scenario, customers are said to perform *social learning* [1] since they learn about an unknown state (the quality of the product) from either the *social observations* (reviews of previous customers) or the *personal evidence* (doing some research by themselves). *Information cascade* [2]–[4] characterizes the situation where rational agents performing social learning are overloaded with social observations so that they can make decisions without considering the personal evidence collected on their own. An information cascade can occur such that customers tend to buy the product when there are many positive reviews in the system. Conversely, customers are prone not to buy the product when there are considerable negative reviews. It is reasonable for rational agents to take into account the social observations since previous decisions may contain information about the unknown state. However, from seller's perspective, an information cascade with positive reviews can lead to an increase in sales. Hence, malicious or dishonest sellers may try to steer an information cascade by manipulate the social observations, i.e. flooding the system with biased reviews, forging a high sales number, or hiding true information. If the people selfishly manipulate an information cascade, then the operation of the social system will be in jeopardy. Therefore, how to design a social system to prevent or to reverse a fake information cascade is a fundamental issue in digital society [5].

In the literature on social learning and information cascade, there are two methodologies to model rational decision-making. Bayesian social learning [2], [3], [6]–[8] assumes agents use Bayes rule to update their posterior probability about the unknown state and maximize their own utility. Non-bayesian social learning [9]–[11] considers other deterministic or randomized rule-of-thumb procedures to aggregate agent's decisions. In the previous literature, much effort has been devoted to the asymptotic learning which studies the convergence of agent's decisions to the true state when the entire population grows to infinity. Recent research directions pay attention to the design issues of a social system with respect to the network externality [12] and the incentive mechanism affecting user behavior in a social system [13].

In this paper, our main purpose is to study the design considerations of steering information cascades in a social system where agents perform Bayesian social learning and make sequential decisions about an unknown binary state.

The main contributions of this paper are listed as follows:

- We explain the key factors that can affect the decisions of Bayesian agents, and summarize the factors as the social system designer's inequalities.
- We generalize and give a precise definition of information cascade in the existing literature on social learning, and apply the general definition to study its behavior in different random network topologies, particularly Erdös-Rényi (ER) networks, small-world networks, and scale-free networks.
- We propose the selective rewiring algorithm and show that by selectively rewiring agent to observe previous

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correct decisions and selectively eliminating wrong decisions in agent's social observations, the overall system performance will be well-steered.

- We propose the incentive seeding algorithm that makes use of utility function as incentives, and show that the system designer can steer information cascades by providing incentives to influential agents in a social system.

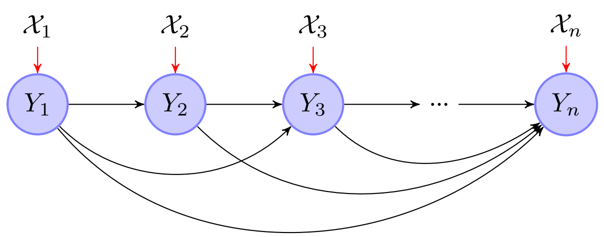



Fig. 1. The Sequential Bayesian Social Learning Model. The red arrows mean that agent $i$ can obtain a set of personal evidence $\mathcal{X}_i$ at time $t = i$. The black arrows illustrate the observation network topology of the social system. A link pointing from $Y_j$ to $Y_i$ means that agent $i$ can observe agent $j$'s decision. This figure shows an example of full observation network, where agent $i$ can observe all of previous decisions $Y_1, Y_2, ..., Y_{i-1}$. We emphasize that more general network topologies are allowed in our model, which means that some of the links in this figure can be taken out to form different observation network topologies, such as Erdös-Rényi networks, small-world networks, and scale-free networks.

## II. System Model and Problem Formulation

To understand agents' decision-making and reaction to incentives in a socially networked system, we model the social system as a sequential decision system in which agents perform social learning and make decision according to a given time order. On the basis of [2], [3], sequential decision is the criterion that information cascade can possibly happen. Consider a set of agents $N = \{1, 2, 3, ..., n\}$ where $n$ is the total number of agents. Each agent has to make a decision about an unknown binary state $\theta \in \{0, 1\}$ sequentially with the time index $t = 1, 2, 3, ..., n$. That is, at time instance $t = i \in N$, agent $i$ makes a decision $Y_i$. Agent $i$'s decision is based on personal evidence $\mathcal{X}_i$ (also known as private signal in some literatures [2], [8], [14]) and/or social observation $\mathcal{Y}_i$, where $\mathcal{X}_i$ is a set of $m$ i.i.d. samples $X_i^{(1)}, X_i^{(2)}, ..., X_i^{(m)}$ from probability distribution $f(x|\theta)$, and $\mathcal{Y}_i$ is the set of previous agents' decisions that can be observed by agent $i$ given the observation network topology $\mathcal{G}$, which can be represented by an adjacency matrix $A_{n\times n} = [a_{ji}]$, with $a_{ji} = 1$ if $i$ can observe $j$, otherwise $a_{ji} = 0$. Then, $\mathcal{Y}_i$ is defined as $\mathcal{Y}_i = \{Y_j | 1 \le j \le i-1 \text{ and } a_{ji} = 1\}$. Figure 1 shows a standard network topology in which agent $i$ can observe all of previous decisions $Y_1, Y_2, ..., Y_{i-1}$ while we do not require this assumption in this paper. After making the decision $Y_i$, agent $i$ will get utility $u_i$. We assume agents are Bayesian and their goals is to maximize their individual utility. The decision process can be precisely formulated as a sequential binary hypothesis testing problem, with the two hypotheses:

$$\mathcal{H}_0 : \theta = 0 \text{ versus } \mathcal{H}_1 : \theta = 1 \tag{1}$$

Agents have priors $P(\mathcal{H}_0) = \pi_0$ and $P(\mathcal{H}_1) = \pi_1$. The distributions of sample data under each hypothesis are

$$X_i^{(j)} \underset{\mathcal{H}_k}{\overset{i.i.d.}{\sim}} f(x|\theta = k) \,\forall k \in \{0, 1\}, i \in N, 1 \le j \le m.$$

For agent $i$, all available information are the sets $\mathcal{X}_i$ and $\mathcal{Y}_i$. We call $\mathcal{I}_i = (\mathcal{X}_i, \mathcal{Y}_i)$ the information set of agent $i$ and define the decision rule $\delta_i$ of agent $i$ as a function $\delta_i : (\mathcal{X}_i, \mathcal{Y}_i) \to \{0, 1\}$ mapping from the information set to the state space. The utility gained from using decision rule $\delta_i$ is $U_i(\delta_i, \theta)$, where $U_i$ is the utility function of agent $i$. Note that $U_i$ can be a function of other variables in the system besides $\delta_i$ and $\theta$.

Then, agents' utility maximization problem is written as

$$\max_{\delta_i} \mathbb{E}\left[U_i(\delta_i, \theta) | (\mathcal{X}_i, \mathcal{Y}_i)\right], \forall i \in N. \tag{2}$$

Solving the optimization problem in (2) involves designing a sequence of decision rules $\delta_1, \delta_2, ..., \delta_n$ that conform to certain optimality criterion which depends on the choice of using Bayesian or non-Bayesian methodologies to aggregate information. We adopt Bayesian methodology here, but our approach is different from the conventional Bayesian approach. Instead of only assuming that the agents know the distributions of sample data under each hypothesis $f(x|\theta = k) \,\forall k \in \{0, 1\}$, we amend an additional assumption that the distributions of agents' decisions under each hypothesis $P_{\theta=k}(\mathcal{Y}_i) \,\forall k \in \{0, 1\}$ can be calculated by all agents. Then, following the Bayes decision criterion, we can solve the optimization problem by sequential likelihood ratio test procedure as follows.

Agent $i$ defines the likelihood ratio test statistic as

$$\mathcal{L}_i(\mathcal{X}_i, \mathcal{Y}_i) = \frac{P_{\theta=1}(\mathcal{X}_i, \mathcal{Y}_i)}{P_{\theta=0}(\mathcal{X}_i, \mathcal{Y}_i)}. \tag{3}$$

Since $\mathcal{X}_i$ is the set of new sample data at time $t = i$ and $\mathcal{Y}_i$ contains a subset of past decisions up to time $t = i-1$, hence $\mathcal{X}_i$ and $\mathcal{Y}_i$ are independent, and we have

$$\begin{aligned}\mathcal{L}_i(\mathcal{X}_i, \mathcal{Y}_i) &= \frac{P_{\theta=1}(\mathcal{X}_i)}{P_{\theta=0}(\mathcal{X}_i)} \frac{P_{\theta=1}(\mathcal{Y}_i)}{P_{\theta=0}(\mathcal{Y}_i)} \\ &= \left[\prod_{j=1}^{m} \frac{f\left(X_i^{(j)}|\theta = 1\right)}{f\left(X_i^{(j)}|\theta = 0\right)}\right] \frac{P_{\theta=1}(\mathcal{Y}_i)}{P_{\theta=0}(\mathcal{Y}_i)}.\end{aligned} \tag{4}$$

Thus, the Bayes optimal decision rule $\delta_i^*$ for agent $i$ given the information set $(\mathcal{X}_i, \mathcal{Y}_i)$ is

$$\delta_i^*(\mathcal{X}_i, \mathcal{Y}_i) = \begin{cases} 1 \text{ , if } \mathcal{L}_i(\mathcal{X}_i, \mathcal{Y}_i) \ge \tau_i \\ 0 \text{ , otherwise} \end{cases}, \tag{5}$$

where $\tau_i = \frac{\pi_0(U_{00} - U_{10})}{\pi_1(U_{11} - U_{01})}$ is the Bayes decision threshold, and $U_{jk} = U_i(\delta_i = j, \theta = k)$ is the utility gained from choosing $\delta_i = j$ when the true state is $\theta = k$.

We can express the decision rule more compactly as

$$\mathcal{L}_i(\mathcal{X}_i, \mathcal{Y}_i) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau_i. \tag{6}$$

From (4), define $L_i(\mathcal{X}_i) = \prod_{j=1}^{m} \frac{f\left(X_i^{(j)}|\theta=1\right)}{f\left(X_i^{(j)}|\theta=0\right)}$, $\Lambda_i(\mathcal{Y}_i) = \frac{P_{\theta=1}(\mathcal{Y}_i)}{P_{\theta=0}(\mathcal{Y}_i)}$, $\beta = \pi_0/\pi_1$, and $\mathcal{U} = \frac{U_{00}-U_{10}}{U_{11}-U_{01}}$, then (6) can be rearranged as

$$L_i(\mathcal{X}_i)\,\Lambda_i(\mathcal{Y}_i) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \beta\,\mathcal{U} \tag{7}$$

We name (7) as the **social system design inequalities** since it can be clearly seen from (7) that there are four main factors which can affect the Bayes decision. For example, increases in $L_i(\mathcal{X}_i)$ and $\Lambda_i(\mathcal{Y}_i)$, or decreases in $\beta$ and $\mathcal{U}$ will make agent $i$ incline to choose $\mathcal{H}_1$. Intuitively, when there exists enough evidence that suggests $\mathcal{H}_1$ is the true hypothesis, or there are a number of previous decisions choosing $\mathcal{H}_1$, it is more reasonable for the agent to make decision $\mathcal{H}_1$, but the decision can be different if the agent has strong initial bias $\beta$ in hypothesis $\mathcal{H}_0$ or the incentive structure $\mathcal{U}$ reward the choice of $\mathcal{H}_0$. We explain the possible realizations of the four factors in a social system as follows.

- Personal Evidence ($\mathcal{X}_i$): Agents can make more accurate decisions when the environment has high signal quality, low uncertainty, which means that the data distributions under each hypothesis are significantly different, or agents are allowed to get a large sample of data.
- Social Observation ($\mathcal{Y}_i$): The reason why social observations affect agent's decision-making can be explained in three ways. First, the assumption on the distributions $P_\theta(\mathcal{Y}_i)$ in our model can be viewed as agent's prior experience or anticipation about how other agents will react in this situation. Second, the sets $\mathcal{Y}_i$ for all $i \in N$ represent the observation network topology of agents. Third, if an agent's decision can change the utility of other agents, then this factor can be interpreted as social influence, peer pressure, or externalities.
- Bias ($\beta$): This factor can be viewed as agent's initial opinion about the unknown state, it can also be interpreted as agent's stubbornness or conservativeness in switching to the other decision.
- Utility or Incentive Structure ($\mathcal{U}$): Utility can represent an agent's preference for the two states. However, from a system engineer's perspective, it is more convenient to interpret this factor as the incentive structure in a social system, and utility-maximizing agents are forced to react in accordance with the incentive structure in order to get better off.

These factors play crucial roles in the engineering of a social system, and can steer information cascades or help agents make better decisions.

### *A. Information Cascade*

When a group of agents form an information cascade, the set of personal evidence has no effect on their Bayes optimal decisions.

**Definition 1.** *A set of agents $C \subset N$ is said to form an information cascade if $\delta_i^*(\mathcal{X}_i, \mathcal{Y}_i) = \delta_i^*(\mathcal{Y}_i)$ where $\delta_i^*$ satisfies*

$$\mathbb{E}\left[U_i(\delta_i^*, \theta)|(\mathcal{X}_i, \mathcal{Y}_i)\right] \geq \mathbb{E}\left[U_i(\delta_i, \theta)|(\mathcal{X}_i, \mathcal{Y}_i)\right] \tag{8}$$

*for all $\delta_i \in \{0, 1\}$ and $i \in C$.*

We say that the set of agents $C$ is an $\mathcal{H}_k$-cascade if $C$ is an information cascade and $\delta_i^* = k$ for all $i \in C$, $k \in \{0, 1\}$.

In order to investigate how one can steer information cascades in a social system, we now derive the conditions of information cascade. By the above definition and (7), if agents $i \in C$ form an information cascade, then without considering their personal evidence, say, $\mathcal{X}_i = \varnothing$, their optimal decisions is given by

$$\Lambda_i(\mathcal{Y}_i) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \beta\,\mathcal{U} \tag{9}$$

Therefore, $\mathcal{H}_1$-cascade occurs if and only if the test statistic satisfies [1]

$$L_i(\mathcal{X}_i)\,\Lambda_i(\mathcal{Y}_i) \geq \beta\,\mathcal{U} \text{ and } \Lambda_i(\mathcal{Y}_i) \geq \beta\,\mathcal{U} \tag{10}$$

Similarly, $\mathcal{H}_0$-cascade occurs if and only if the test statistic satisfies

$$L_i(\mathcal{X}_i)\,\Lambda_i(\mathcal{Y}_i) < \beta\,\mathcal{U} \text{ and } \Lambda_i(\mathcal{Y}_i) < \beta\,\mathcal{U} \tag{11}$$

### *B. Performance Measures of the Social System*

We now care the overall system performance when information cascades occur. To improve the system performance, the following performance measures may be meaningful.

*1) Error Probability $P_E$:* The error probability of the social system is defined by the fraction of wrong decisions.

$$P_E = \frac{\sum_{i=1}^{n} \mathbb{I}_{\{Y_i \neq \theta\}}}{n},$$

where $\mathbb{I}$ is the indicator function of a wrong decision.

*2) Fraction of $\mathcal{H}_k$-cascade:* The fraction of $\mathcal{H}_k$-cascades, $R_k$, is defined as the number of agents that form an $\mathcal{H}_k$-cascade. That is,

$$R_k = \frac{|C_k|}{n},$$

where $|C_k|$ is the size of the set of agents that form an $\mathcal{H}_k$-cascade.

*3) Average Total Utility:* The average total utility is defined as the sum of each agent's utility over the entire population of agents.

$$U_{total} = \frac{\sum_{i=1}^{n} u_i}{n}.$$

[1] Here we do not consider randomized decisions for simplicity. Note that although allowing randomized decisions may improve system performance, it is still possible to steer information cascades by manipulating social observations and utility functions.

## III. Selective Rewiring and Incentive Seeding

In this section, we propose two approaches, selective rewiring and incentive seeding, which can be perform to adjust agents' information structure via controlling the factors in (7), and consequently steer information cascades.

In practice, it is hard to identify which factors in (7) are controllable in a socially networked system. For instance, in the scenario of a customer review system discussed in Section I, customer's personal evidence $\mathcal{X}_i$, bias $\beta$, and preference in utiltiy $\mathcal{U}$ are not possible to be controlled by the system designer and can only be estimated from experiments. Nevertheless, it is possible for the system to change the social observation $\mathcal{Y}_i$ and incentive structure $\mathcal{U}$, i.e. the system can decide which subset of the reviews to be shown to customers, or provide good customer reviews with rewards. On the assumption that the system can change $\mathcal{Y}_i$, the intuition behind the proposed selective rewiring approach is that in order to steer an $\mathcal{H}_k$-cascade, we can selectively rewire agent $i$'s observation network to connect previous decisions $Y_j = k$ or to disconnect previous decisions $Y_j \neq k, 1 \leq j \leq i-1$. On the other hand, assuming that $\mathcal{U}$ can be changed by the system, the intuition behind the proposed incentive seeding approach is that we first identify the influential agents in the system, and then provide them with incentives to make decision $\mathcal{H}_k$ in order to steer an $\mathcal{H}_k$-cascade.

### A. Selective Rewiring

The intuition of selective rewiring algorithm is to restructure an agent's social observation by disconnecting undesirable decisions and connecting desirable decisions. The pseudo code of the proposed selective rewiring algorithm is shown in Algorithm 1. Suppose that the system designer desires to steer an $\mathcal{H}_k$-cascade, then the system designer can iteratively change the social observation set $\mathcal{Y}_i$ for each agent $i \in N$ by excluding a subset $\mathcal{Z}_i \subset \mathcal{Y}_i$ with size $|\mathcal{Z}_i| = N_z$ that contains agents who make decision $z \neq \mathcal{H}_k$, or by including a set $\mathcal{W}_i$ with size $|\mathcal{W}_i| = N_w$ which contains agents who are not observed by agent $i$ and make the decision $w = \mathcal{H}_k$.

**Algorithm 1** Selective Rewiring

**Input:** $N_z, N_w, \mathcal{H}_k$
**for** $i = 1$ **to** $n$ **do**
  Find $\mathcal{Z}_i = \{z | z \neq \mathcal{H}_k\}$, $\mathcal{Z}_i \subset \mathcal{Y}_i$ and $|\mathcal{Z}_i| = N_z$
  Find $\mathcal{W}_i = \{w | w = \mathcal{H}_k\}$, $\mathcal{W}_i \cap \mathcal{Y}_i = \varnothing$ and $|\mathcal{W}_i| = N_w$
  $\mathcal{Y}_i \leftarrow (\mathcal{Y}_i \backslash \mathcal{Z}_i) \cup \mathcal{W}_i$
**end for**

### B. Incentive Seeding

Let us consider the incentive seeding approach when the system designer can change the incentive structure $\mathcal{U}$. The word "seeding" means that the system designer can spread some rewards, i.e. bonus, coupon, or discount, etc., in the system. Hence, to make use of the observation network structure, the system designer first identifies the agents whose out-degree are greater than a specified minimum value $d_{min}^{out}$. Then, in order to steer an $\mathcal{H}_k$-cascade, those agents are provided with incentives $U^*$ which make them in favor of making decision $\mathcal{H}_k$. The pseudo code of the proposed incentive seeding algorithm is shown in Algorithm 2.

**Algorithm 2** Incentive Seeding

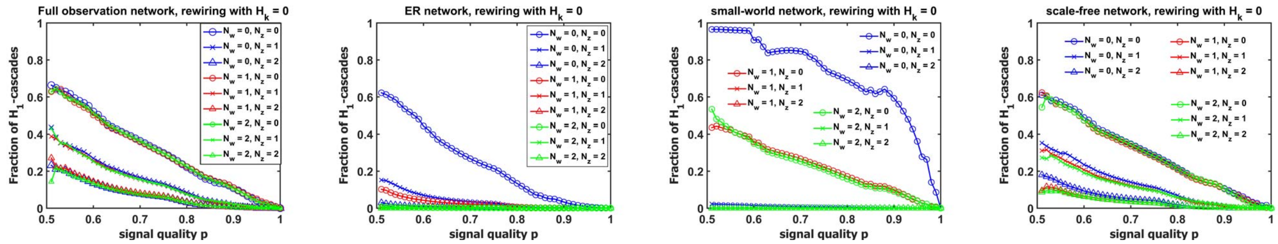
**Input:** $d_{min}^{out}, U^*, \mathcal{H}_k$
**for** $i = 1$ **to** $n$ **do**
  **if** $d_i^{out} \geq d_{min}^{out}$ **then**
    $U_i(k,k) \leftarrow U^*$, where $U_i(k,k) < U^*$
  **end if**
**end for**

## IV. Numerical Examples and Interpretations

In this section, we consider two numerical examples that illustrate the effect due to selective rewiring and incentive seeding. In the following numerical examples, we let agents' prior distribution about $\theta$ be equally likely, $\pi_0 = \pi_1 = 1/2$. We assume $m = 1$ for each agent and that the sample data under each hypothesis are Bernoulli distributed with parameter $p$ and $1-p$ under hypothesis $\mathcal{H}_1$ and $\mathcal{H}_0$ respectively. So, $f(x|\theta=1) = p^x (1-p)^{1-x}$ and $f(x|\theta=0) = (1-p)^x p^{1-x}$, $x \in \{0,1\}$, where $1/2 < p < 1$ is the signal quality which can be interpreted as the accuracy of agent's personal evidence in the inference of $\theta$. Moreover, to simplify the complex interactions among agents' decisions, we consider the simplest case where agents using their social observations in the Bayes update rule according to their own personal evidence. Therefore, we assume that agents' prior distributions of social observations under each hypotheses are Bernoulli distribution with paramter $p$ and $1-p$ under $\mathcal{H}_1$ and $\mathcal{H}_0$ respectively. So, $P_{\theta=1}(y) = p^y (1-p)^{1-y}$ and $P_{\theta=0}(y) = (1-p)^y p^{1-y}$ for $Y_j = y \in \{0,1\}$ and $Y_j \in \mathcal{Y}_i$. This assumption on the distribution corresponds to the naïve Bayes approach, which means that agents treat social observations as independent data in the updating of their posterior distribution about $\theta$. It is noteworthy that this assumption can be relaxed when applying the model to real-world scenarios.

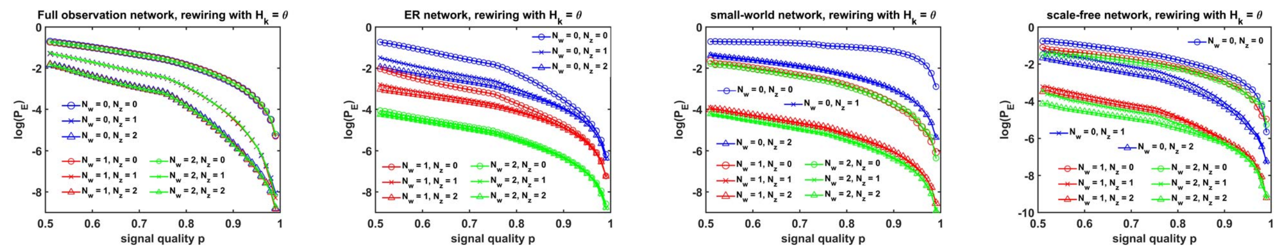
### A. Information Cascades by Selective Rewiring

In the first numerical example, we consider systems of $n = 100$ agents with full observation of earlier decisions, ER, small-world, and scale-free network topologies. The full observation network is as described in Figure 1. The ER network is generated with probability 0.1 for each pair of nodes to connect. The small-world network is generated with each node connecting to one nearest neighbor and with probability 0.1 to reconnect to another node. The scale-free network is generated with minimum node degree 10 using preferential attachment in [15]. Each random network topology of the same type has approximately the same mean degree. The utility of agents are set as $U_{11} = U_{00} = 1$, and $U_{10} = U_{01} = 0$, and system designer can see the realization of $\theta$ and perform selective rewiring with input $N_z, N_w, \mathcal{H}_\theta$ to change agents' social observations. It is of interest from a system design perspective to know the




(a) Full observation network (b) ER network (c) Small-world network (d) Scale-free network

Fig. 2. Fraction of $\mathcal{H}_1$-cascades in different observation network topologies when selective rewiring is performed with $k = 0$. For each figure (a)-(d), the vertical axis is the fraction of $\mathcal{H}_1$-cascades defined as $R_1 = \frac{|C_1|}{n}$ in Section II-B, and the horizontal axis is the signal quality $0.5 < p < 1$. The fraction of $\mathcal{H}_1$-cascades decreases when $p$ increases since we let $\theta = 0$ in this numerical example.




(a) Full observation network (b) ER network (c) Small-world network (d) Scale-free network

Fig. 3. Error probability $log\,(P_E)$ in different observation network topologies when selective rewiring is performed with $k = \theta$. The error probability is $P_E = \frac{\sum_{i=1}^{n} \mathbb{I}_{\{Y_i \neq \theta\}}}{n}$ as defined in Section II-B. The vertical axis of (a)-(d) is $log\,(P_E)$, which decreases as $p$ increases. We can see the effect due to selective rewiring with different values of $N_z, N_w$ in each random network topology.

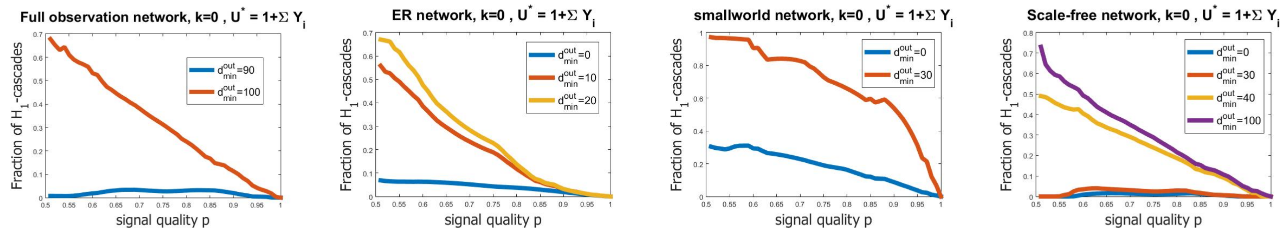
way to set the values of $N_z, N_w$ and their effect on agents' decisions. Hence, assuming that the system designer knows that the true state is $\theta = 0$, Figure 2 and Figure 3 show the results of performing selective rewiring with different values of $N_z, N_w$ in four network topologies with $k = 0$. When selective rewiring is performed ($N_z > 0, N_w > 0$), we can see that the fraction of $\mathcal{H}_1$-cascades and error probability are lowered in comparison with no rewiring ($N_z = N_w = 0$). However, disconnection ($N_z$) and connection ($N_w$) have different effects in different network topologies. In Figure 2 (a) and Figure 3 (a), increasing $N_w$ cannot improve the system performance in the full observation network because the set $\mathcal{W}_i$ is always empty. In Figure 2 (b) and Figure 3 (b), increasing $N_w$ is more effective than increasing $N_z$ on improving system performance in ER network. In Figure 2 (c) and Figure 3 (c), the effect of disconnection saturates when $N_z \geq 1$, so it is better to combine the use of connection and disconnection in small-world network. In Figure 2 (d) and Figure 3 (d), simply using disconnection ($N_z > 0, N_w = 0$) appears more effective than simply using connection ($N_z = 0, N_w > 0$).

**Remark 1.** In order to steer a desirable information cascade by selective rewiring, say, an $\mathcal{H}_0$-cascade in this numerical example, the system designer can use connection and disconnection simultaneously (i.e. $N_z = 1, N_w = 1$), which works for most random network topologies. Moreover, if one attempts to steer the information cascade to a larger fraction of users, then one can use more disconnection (i.e. increase $N_z$) for full observation network. For an ER network, one can amend more links (i.e. increase $N_w$). For small-world and scale-free networks, it is better to combine the use of adding more new links and deleting bad links (i.e. increase $N_z, N_w$ simultaneously) to steer an $\mathcal{H}_0$-cascade.

## B. Information Cascades by Incentive Seeding

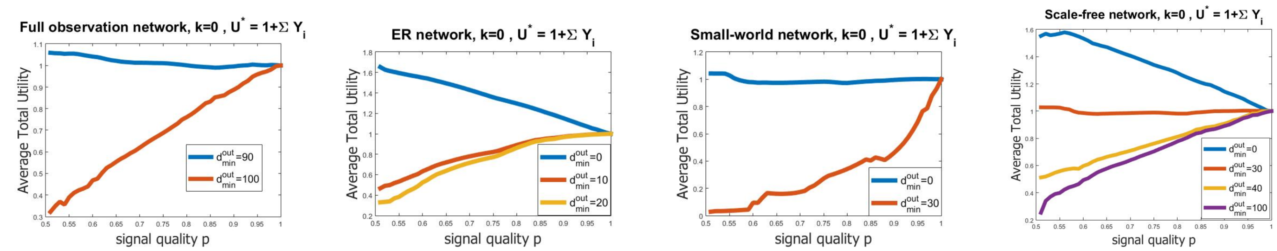
In the second numerical example, we consider the same setting of random networks as Section IV-A, but the system designer performs incentive seeding instead. Since we assume that the true parameter observed by the system designer is $\theta = 0$, the incentive seeding is performed with input $k = 0$, and we need to choose the values of $d_{min}^{out}$ and the function $U^*(\mathcal{Y}_i)$, which depends on social observations. The function $U^*(\mathcal{Y}_i)$ determines how the system gives incentives to agents making decision $\mathcal{H}_0$. Since the true state is $\theta = 0$, we let $U^*$ be a linear increasing function of the number of decisions $\mathcal{H}_1$ in $\mathcal{Y}_i$, that is, $U_i^* = 1 + \sum_{j \in \mathcal{Y}_i} Y_j$, which means that the system rewards agents who make decision $\mathcal{H}_0$ when there are many agents making decision $\mathcal{H}_1$. After the incentive function is chosen, the system designer need to set the value of $d_{min}^{out}$ which determines a set of influential agents to be distributed with the incentives. The results are shown in Figure 4 and 5.

**Remark 2.** In terms of reducing the fraction of $\mathcal{H}_1$-cascade when the signal quality $p$ increases, an ER network is close to full observation, but a small-world network has a slower rate of decreasing and a scale-free network has a faster rate of decreasing. It is because the small-world network has lower value of diameter which is more difficult to change every agent's decision. However, in the scale-free network, there




(a) Full observation network (b) ER network (c) Small-world network (d) Scale-free network

Fig. 4. Fraction of $\mathcal{H}_1$-cascades in different observation network topologies when incentive seeding is performed with $k = 0, U_i^* = 1 + \sum_{j\in\mathcal{Y}_i} Y_j$. The parameter $d_{min}^{out}$ specifies the minimum outdegree of the set of agents who can get rewards by acting in accordance with the desirable information cascade steered by the system designer, i.e. $\mathcal{H}_0$-cascades in this numerical example. We can see that $\mathcal{H}_0$-cascades (substantial decreases in $\mathcal{H}_1$-cascades) is steered when an effective value of $d_{min}^{out}$ is chosen.




(a) Full observation network (b) ER network (c) Small-world network (d) Scale-free network

Fig. 5. Average total utility in different observation network topologies when incentive seeding is performed with $k = 0, U_i^* = 1 + \sum_{j\in\mathcal{Y}_i} Y_j$. For each figure (a)-(d), the vertical axis is $U_{total} = \frac{\sum_{i=1}^{n} u_i}{n}$ as defined in Section II-B. This figure can help us determine which value of $d_{min}^{out}$ is a good choice. For instance, the choices of $d_{min}^{out} = 90$ in (a) and $d_{min}^{out} = 30$ in (d) make the curve of average total utility looks like a constant function $U_{total}(p) = 1$ for all $0.5 < p < 1$, which means that each agent can get utility one on average when an information cascade is steered, and the system designer need not to waste excess incentives on agents. However, note that in (c), $d_{min}^{out} = 0$ is not a good choice since $d_{min}^{out} = 0$ means that the system designer has to give incentives to all agents.

exist agents of very high outdegrees or of very low outdegrees, which is therefore easier to steer an information cascade.

## V. Conclusions

In this paper, the design considerations of a socially networked system are studied through a sequential Bayesian social learning model. We formulate the Bayes decision-making process as a binary sequential hypothesis testing problem and analyze the factors that can affect agents' decisions by the social system design inequalities, and define information cascade in a precise way. Then, we illustrate that different types of random networks make difference in the formation of information cascades. We propose the selective rewiring and incentive seeding algorithms, and demonstrate that the system performance measures can be improved when the algorithms are performed to steer a desirable information cascade.

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